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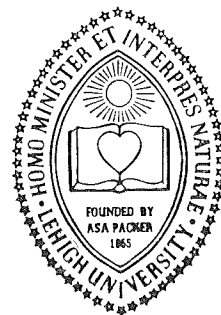
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**Low Cycle Fatigue Behavior of Jointed Structures**

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LABORATORY LIBRARY**

**REDISTRIBUTION OF  
STRESS AND STRAIN  
IN A BEAM WITH A CRACK**

by  
**R. J. Smith  
P. Marek  
B. T. Yen**

**Fritz Engineering Laboratory Report No. 358.9**

Low Cycle Fatigue Behavior of Joined Structures

REDISTRIBUTION OF STRESS AND STRAIN  
IN A BEAM WITH A CRACK

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The work of this progress report was conducted as part of a study of low-cycle fatigue, sponsored by the Office of Naval Research, Department of Defense, under Contract N 00014-68-A-514; NR 064-509. Reproduction in whole or part is permitted for any purpose of the United States Government.

Department of Civil Engineering

Fritz Engineering Laboratory  
Lehigh University  
Bethlehem, Pennsylvania

June 1970

Fritz Engineering Laboratory Report No. 358.9

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ABSTRACT

This report presents an analytical method for determining the stress and strain distribution in a welded I-beam with a three-ended crack. This investigation is part of a major research program on the behavior and design of joined structures subjected to low-cycle fatigue.

The analysis is on the plane-stress elastic-plastic behavior of a beam with a crack for one loading-unloading cycle. A mathematical model was developed by replacing the continuous material of the beam by an equivalent system of lumped volumes at discrete points.

The results of the discrete element method were compared with the stresses and strains recorded in an experimental investigation of a welded beam with a three-ended crack in the last phase of its fatigue life. Satisfactory correlation between measured and theoretical results was observed.

## 1. INTRODUCTION

As part of a study of the low-cycle fatigue behavior of joined structures, a theoretical study has been conducted to obtain the stress and strain redistribution that occurs in an I-beam with a three-ended crack. The lumped parameter method of analysis, which was used in a previous investigation on a cracked plate,<sup>(1)</sup> was formulated to analyze the elasto-plastic behavior of a beam with a progressive crack length under a loading-unloading cycle.

The objective of this report is to describe the formulation of the method and to present the results in conjunction with experimental finding. It is expected that further review and analysis of these results will bring about a better understanding of the low-cycle fatigue phenomenon.

## 2. ANALYTIC FORMULATION

### 2.1 Basic Formulation and Assumptions

The basis of the lumped parameter method for stress analysis consists in approximating the continuous material of the portion of the beam under investigation by a model composed of lumped volumes. These lumped volumes are arranged at discrete points according to a definite pattern or gridwork in the model structure.

The welded I-beam under investigation was subjected to a constant moment,  $M_a$ , resulting in an applied stress distribution,  $\sigma_a$ , as shown in Fig. 1. In this figure,  $L$  represents the length of the beam segment under investigation,  $w$  and  $t$  are the width and thickness of the flanges,  $d$  is the depth of the web and  $t_w$  is its thickness. The beam was doubly symmetric in cross sectional shape and its analysis could be assumed as one of plane-stress.

Certain conditions were imposed on the portion of the beam under investigation in order to define a discrete

model representation of this area. These conditions were to assume the following:

1. The strain  $\epsilon_a$  at sections B-B sufficiently far away from the crack (Fig. 1) varies linearly under the applied stress  $\sigma_a$ . This implies that sections B-B remain plane even though there are elements becoming plastified under  $\sigma_a$ . In addition,  $\epsilon_a$  at B-B is adjusted in order to maintain equilibrium at section A-A where the crack exists.
2. Section A-A is the reference plane at which no displacement occurs.
3. The crack at section A-A starts at the center of the tension flange and propagates into the web and symmetrically across the flange, as illustrated by the shaded area of Fig. 1.
4. There is no change in the position of the neutral axis of the beam due to either



plastification of elements or the presence of the crack. There is no longitudinal strain or elongation at the neutral axis of the beam.

5. The compression portion of the beam under investigation is not considered in the analysis.

Considering the symmetry of the applied stress, of the residual stress pattern, of the crack and of the beam geometry, the analysis could be formulated for one quadrant of the beam segment. This dimensional region is shown in Fig. 2A.

The discrete model representation of this beam quadrant is shown in Fig. 2B. It consists of rectangular elements lumped to points in a rectangular gridwork that lies on the middle planes of the flange and half of the web plate. These lumped parameters are represented by the solid circles of Fig. 2B. In this figure  $L'$ ,  $w'$ , and  $d'$  signify distances between lumped parameters in the grid

system and  $D$  denotes the depth of the web plate considered for discrete model representation. The model was divided into an arbitrary number of mesh divisions  $M \times Z$  for the flange and  $N \times P$  for the web. The mesh divisions  $Z$  and  $P$  must be equal.

Other assumptions made in the analysis were:

1. The effect of Poisson's ratio was neglected
2. The shear modulus,  $G$ , was constant throughout the elastic and the plastic strain range.
3. The stress-strain curve was bi-linearly elastic-strain hardening, and
4. The applicable yield condition was the maximum normal stress (Rankine) theory.

## 2.2 Formulation of Equilibrium Equations

The conventional stiffness method of structural analysis was applied to establish the equilibrium equations from the relative displacements of the lumped parameters. A typical displacement configuration of the lumped parameters is shown in Fig. 3. The stress resultants or forces that act on two discrete points  $I$  and  $J$  at the

flange-web intersection are represented in Fig. 4 where  $V_I$  and  $V_J$  are the shear stress resultants,  $R_I$  and  $R_J$  are the residual stress resultants and  $S_I$  and  $S_J$  are the normal stress resultants for the flange and for the web respectively. The formulation of the equilibrium equations is similar to that for the plate described in detail in Ref. 1.

Three conditions of equilibrium are possible within the discrete model, depending upon the location of the nodal point and its state of stress. These are the elastic, the plastic or the cracked condition of equilibrium.

I. For the elastic condition, the following equilibrium equations may be written:

A. In the flange:

a. For nodes on longitudinal grid lines 2 to M-1.

$$S_I - S_{I-M} + V_I - V_{I+1} + R_I - R_{I-M} = 0 \quad (1)$$

The residual stress resultants  $R_{I-M}$  and  $R_I$  are identical at a nodal point thus can be dropped from Eq. (1). The remaining forces are substituted by the products of relative displacements and stiffness coefficients.

$$(X_I - X_{I+M}) C_I + U_I - (X_{I-M} - X_I) C_{I-M} - U_{I-M} + (X_I - X_{I-1}) D_I - (X_{I+1} - X_I) D_I = 0 \quad (2)$$

where  $C_I$  and  $D_I$  are the elastic stiffness coefficients for tension and shear in the flange, respectively,

$$C_I = \frac{Etw (2Z-1)}{2ML} \quad (3)$$

$$D_I = \frac{2GtLM}{(2Z-1)w} \quad (4)$$

The  $U$ 's are forces resulting from yielding thus are zero in the elastic range. Equation (2) can be rearranged as:

$$\begin{aligned} X_{I+M} (-C_I) + X_{I+1} (-D_I) + X_I (C_I + C_{I-M} + 2D_I) \\ + X_{I-1} (-D_I) + X_{I-M} (-C_{I-M}) = U_{I-M} - U_I \end{aligned} \quad (5)$$

b. For nodes on longitudinal grid line  $M$ ,

By substituting for the stress resultants in Eq. (1), the following expression is obtained

$$\begin{aligned} (X_I - X_{I+M}) C_I + U_I - (X_{I-M} - X_I) C_{I-M} - U_{I-M} \\ + (X_I - X_{I-1}) D_I = 0 \end{aligned} \quad (6)$$

or

$$\begin{aligned}
 & X_{I+M} (-C_I) + X_I (C_I + C_{I-M} + D_I) \\
 & + X_{I-1} (-D_I) + X_{I-M} (-C_{I-M}) = U_{I-M} - U_I
 \end{aligned} \quad (7)$$

c. For nodes on longitudinal grid line 1,

Equilibrium requires that:

$$S_I - S_{I-M} + V_I - V_{I+1} + R_I - R_{I-M} + V_J = 0 \quad (8)$$

Expressed in terms of relative displacements and stiffness coefficients, the following equation is obtained

$$\begin{aligned}
 & (X_I - X_{I+M}) C_I + U_I - (X_{I-M} - X_I) C_{I-M} - U_{I-M} \\
 & + (X_I - X_{I-1}) D_I - (X_{I+1} - X_I) D_I + [(X_I - X_{IC}) \\
 & - (X_J - X_{JC})] D_J = 0
 \end{aligned} \quad (9)$$

Since  $X_I = X_{I-1}$  by symmetry, Eq. (9) can be written as:

$$\begin{aligned}
 & X_{I+M} (-C_I) + X_{I+1} (-D_I) + X_I (C_I + C_{I-M} + D_I + D_J) \\
 & + X_{I-M} (-C_{I-M}) + (X_{IC} + X_J - X_{JC}) (-D_J) = \\
 & U_{I-M} - U_I
 \end{aligned} \quad (10)$$

B. For the web:

Equilibrium requires that

$$S_J - S_{J-N} + V_{J+1} - V_J + R_J - R_{J-N} = 0 \quad (11)$$

The residual stress resultants  $R_J$  and  $R_{J-N}$  are identical at a nodal point thus can be dropped from Eq. (11).

a. For nodes on longitudinal grid lines 2 to N-1,

By substituting the stress resultants by displacements and stiffness coefficients, and rearranging, Eq. (11) becomes

$$\begin{aligned} &X_{J+N} (-C_J) + X_{J+1} (-D_J) + X_J (C_J + C_{J-N} + 2D_J) \\ &+ X_{J-1} (-D_J) + X_{J-N} (-C_{J-N}) = U_{J-N} - U_J \end{aligned} \quad (12)$$

where  $C_J$  and  $D_J$  are the elastic stiffness coefficients for tension and shear in the web, respectively,

$$C_J = \frac{E t_w (d+t) (2P-1)}{4 (N+1) L} \quad (13)$$

$$D_J = \frac{G t_w L (N+1)}{(2P-1) (d+t)} \quad (14)$$

The U's are forces resulting from yielding thus are zero in the elastic range.

b. For nodes on longitudinal grid line N,

$$\begin{aligned} & X_{J+N} (-C_J) + X_J (C_J + C_{J-N} + D_J) \\ & + X_{J-1} (-D_J) + X_{J-N} (-C_{J-N}) = U_{J-N} - U_J \end{aligned} \quad (15)$$

c. For nodes on longitudinal grid line 1,

$$\begin{aligned} & (X_J - X_{J+N}) C_J + U_J - (X_{J-N} - X_J) C_{J-N} - U_{J-N} \\ & + [(X_J - X_{JC}) - (X_{J+1} - X_{JC+1})] D_J \\ & - [(X_I - X_{IC}) - (X_J - X_{JC})] D_J = 0 \end{aligned} \quad (16)$$

or

$$\begin{aligned} & X_{J+N} (-C_J) + X_{J+1} (-D_J) + X_J (C_J + C_{J-N} + 2D_J) \\ & + X_{J-N} (-C_{J-N}) + (X_I - X_{IC} + 2X_{JC} - X_{JC+1}) \\ & (-D_J) = U_{J-N} - U_J \end{aligned} \quad (17)$$

II. For the yielded condition of any element, the stiffness coefficients  $C_I$  and  $C_J$  are modified and the yielding induced forces  $U_I$  and  $U_J$  are generated. The coefficient changes are represented by a substitution of the strain-hardening modulus,  $E_s$ , for the modulus of elasticity,  $E$ , in Eqs. (3) and (13)

$$C_I = \frac{E_s t w (2Z-1)}{2ML} \quad (18)$$

$$C_J = \frac{E_s t_w (d+t) (2P-1)}{4 (N+1) L} \quad (19)$$

The forces  $U_I$  and  $U_J$  are:

$$U_I = \frac{F_y t w}{2M} - R_I \quad (20)$$

$$U_J = \frac{F_y t_w (d+t)}{4 (N+1)} - R_J \quad (21)$$

where

$$R_I = \frac{t w S_{res}}{2M} \quad (22)$$

and

$$R_J = \frac{t_w (d+t) S_{res}}{4 (N+1)} \quad (23)$$

Since the shear modulus,  $G$ , was assumed to be constant in the analysis, the stiffness coefficients  $D_I$  and  $D_J$  remain the same as in the elastic case.

III. For the elements adjacent to the crack, the stiffness coefficients  $C_I$  and  $C_J$  are zero and the forces  $U_I$  and  $U_J$  become:

$$U_I = -R_I \quad (24)$$

$$U_J = -R_J \quad (25)$$



The above equilibrium equations (Eqs. 5, 7, 10, 12, 15, and 17) can be written in generalized form. For the flange,

$$\begin{aligned} & AA_I (X_{I+M}) + BB_I (X_{I+1}) + CC_I (X_I) + DD_I (X_{I-1}) \\ & + EE_I (X_{I-M}) + GG_I (X_J + X_{IC} - X_{JC}) = FF_I \end{aligned} \quad (26)$$

For the web,

$$\begin{aligned} & AA_J (X_{J+N}) + BB_J (X_{J+1}) + CC_J (X_J) + DD_J (X_{J-1}) \\ & + EE_J (X_{J-N}) + GG_J (X_I - X_{IC} + 2X_{JC} - X_{JC+1}) \\ & = FF_J \end{aligned} \quad (27)$$

### 2.3 Solution Procedure

The solution of Eqs. 26 and 27 employed the Gauss-Seidel iterative approach.<sup>(2)</sup> The solution procedure, carried out by the use of a CDC 6400 computer, was incremental as well as iterative. An explanation of the incremental approach has been summarized in a previous report.<sup>(1)</sup>

A major problem encountered in the investigation of the 3-ended crack was to characterize the interaction or

shear transfer that occurs at the web-flange connection. The model employed in this analysis used twice a small region of the web-flange connection, once as part of the lumped volume in the flange and then as part of the lumped volume in the web. This region is shown shaded in Fig. 2A. The area of this region is small and can be assumed to represent the contribution of the fillet at the flange and web junction.

### 3. RESULTS AND DISCUSSION

The results of the analytical method have been compared to those of an experimental fatigue crack growth test on a welded beam.<sup>(3)</sup> The material properties, the initial residual stress pattern and the nominal applied stresses used in the computer program of this analytical study corresponded to those of the experimental investigation. Therefore, a direct comparison of the stress and strain redistribution could be made.

The theoretical stress distributions in a cracked beam under load ( $\sigma_{\max} = 36.2$  ksi) and after unloading ( $\sigma_{\min} = 0$  ksi) for different crack sizes are plotted in Fig. 5 for the flange and in Fig. 6 for the web. The dashed lines marked  $S_{\text{res}}$  represent the initial residual stress pattern. The other curved lines in the figures represent the theoretically obtained values and the plotted points (open and closed dots) represent the results of the experiment. The theoretical curves under load were truncated at the yield stress which was 110 ksi. The

distribution curves at first application of load (zero cycle) were used as references for evaluation of experimental stresses from strains recorded at subsequent crack lengths. The analytical and experimental stresses compare satisfactorily.

The results of analysis indicated that for equilibrium at the cracked section there may be transfer of forces from the web to the flange, or vice versa. The transfer of force would occur through shearing action at the web-flange junction in the segment of the beam being affected by the presence of the crack. The computed shear transfer from the web to the flange after unloading is shown in Table 1 for increasing lengths of crack in the beam. The number of cracked elements in the analysis were proportional to but not exactly the same as the crack lengths observed in the experiment. The crack length deviations along with the presence of residual stresses and the many assumptions made in the analysis may have produced the theoretical shear transfer. Further investigation into this aspect should be conducted.

The variation of maximum and minimum strains in the element immediately in front of the crack was evaluated

and is plotted in Fig. 7 for the flange and in Fig. 8 for the web, respectively. After crack initiation, the strains increased very rapidly due to the release of residual tension stress. As the crack length increased, these strains decreased while the strain range,  $\Delta\epsilon_T$ , increased slightly in the web and at a higher rate in the flange. At large crack lengths, the strains and strain range in the flange increased rapidly as a result of the greatly reduced net flange area. However, in the web the change was slow because the crack propagated towards regions of lower applied stresses.

Because of the relatively large size of the discrete elements and of the simplifying postulations employed in the computer program, the computed strains cannot be considered accurate. However, the qualitative variation of the strains during different phases of crack propagation may be significant in relation to the observed behavior of the experimental investigation. The analytically obtained strain history relates fairly well to the recorded surface crack growth during the experiment. This correlation can be seen for the flange in Fig. 9 which is a plot of the half crack length,  $a$ , and strain

range,  $\Delta\epsilon_T$ , versus the number of cycles. In the early stage of the test, the flange surface crack length increased in a linear manner with the number of applied cycles, as did the strain range computed for the element immediately in front of the crack tip. Late in the fatigue life, the crack length and the strain range increased at an accelerating rate up to final failure. Further study and correlation of the strain history with observed crack propagation rates should be conducted.

#### 4. SUMMARY AND CONCLUSIONS

The investigation described in this report used the lumped parameter method to analyze the stress and strain distribution in an I-beam with a three-ended crack. The material properties, the initial residual stress pattern, the magnitudes of the applied stresses and the crack lengths in the flange and the web were, respectively, those of an experimental beam under low-cycle fatigue.<sup>(3)</sup> The following conclusions are drawn from the study:

1. The results of comparison between the stresses from the analysis and from the experimental observations was satisfactory.
2. Qualitatively, the analytical strain history related fairly well with the recorded surface crack growth.
3. Further investigation should be conducted to correlate quantitatively strains and

strain ranges in front of the crack tip  
with crack propagation rates.



## 5. ACKNOWLEDGEMENTS

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6. NOMENCLATURE

|       |   |
|-------|---|
| $C_I$ | Stiffness coefficient for tension in flange<br>(kip./in.)                             |
| $C_J$ | Stiffness coefficient for tension in web (kip./in.)                                   |
| $D_I$ | Stiffness coefficient for shear in flange (kip./in.)                                  |
| $D_J$ | Stiffness coefficient for shear in web (kip./in.)                                     |
| $d$   | Depth of web (in.)  |
| $D$   | Depth of web plate considered for discrete model<br>representation (in.)              |
| $d'$  | Transverse distance between longitudinal grid lines<br>on web model (in.)             |
| $E$   | Young's Modulus (ksi)   |
| $E_s$ | Strain-hardening modulus (ksi)  |
| $F_y$ | Static yield strength (ksi)   |
| $G$   | Shear modulus (ksi)   |
| $L$   | Length of beam segment under investigation (in.)                                      |
| $L'$  | Longitudinal distance between transverse grid<br>lines on flange and web models (in.) |
| $M$   | Number of grid lines or divisions across flange<br>model width                        |

|           |  |
|-----------|--|
| $M_a$     | Applied moment   |
| $N$       | Number of grid lines or divisions across web<br>model depth  |
| $P$       | Number of grid lines or divisions along web<br>model length  |
| $R_I$     | Residual stress resultant in flange (kips)   |
| $R_J$     | Residual stress resultant in web (kips)  |
| $S_I$     | Normal stress resultant in flange (kips)   |
| $S_J$     | Normal stress resultant in web (kips)  |
| $S_{res}$ | Initial residual stress (ksi)  |
| $t$       | Thickness of flange (in.)  |
| $t_w$     | Thickness of web (in.)   |
| $U_I$     | Force on element in flange at yielding (kips)  |
| $U_J$     | Force on element in web at yielding (kips)   |
| $V_I$     | Shear stress resultant in flange (kips)  |
| $V_J$     | Shear stress resultant in web (kips)   |
| $w$       | Width of flange (in.)  |
| $w'$      | Transverse distance between longitudinal grid<br>lines on flange model width (in.)                                   |
| $x_I$     | Displacement of lumped parameter in flange from<br>initial position in discrete model (in.)                          |
| $x_{IC}$  | Constant displacement of lumped parameter in flange<br>which varies linearly with distance from section<br>A-A (in.) |

|                    |  |
|--------------------|--|
| $X_J$              | Displacement of lumped parameter in web from initial position in discrete model (in.)  |
| $X_{JC}$           | Constant displacement of lumped parameter in web which varies linearly with distance from section A-A and also with distance from neutral axis (in.) |
| $Z$                | Number of grid lines or divisions along flange model length  |
| $\epsilon$         | Strain (in./in.)   |
| $\epsilon_a$       | Applied strain (in./in.)   |
| $\epsilon_{max}$   | Maximum strain (in./in.)   |
| $\epsilon_{min}$   | Minimum strain (in./in.)   |
| $\Delta\epsilon_T$ | Strain range ( $\epsilon_{max} - \epsilon_{min}$ ) (in./in.)   |
| $\sigma$           | Stress (ksi)   |
| $\sigma_a$         | Applied normal stress (ksi)  |
| $\sigma_{max}$     | Maximum stress (ksi)   |
| $\sigma_{min}$     | Minimum stress (ksi)   |

7. TABLE AND FIGURES

TABLE 1 FORCE TRANSFERRED FROM WEB TO FLANGE

| Number of Cracked Elements |     | Force Transferred (kips) |
|----------------------------|-----|--------------------------|
| Flange                     | Web | After Unloading          |
| 0                          | 0   | 0                        |
| 2                          | 1   | .9802                    |
| 4                          | 2   | 2.0417                   |
| 6                          | 3   | 2.9550                   |
| 8                          | 4   | 3.5551                   |
| 10                         | 5   | 3.9511                   |
| 12                         | 6   | 4.0581                   |

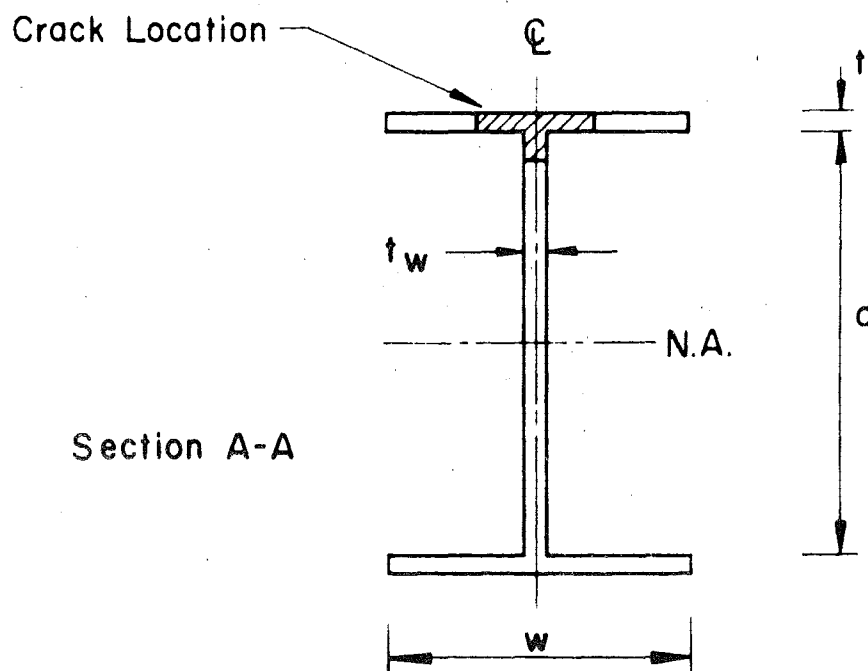
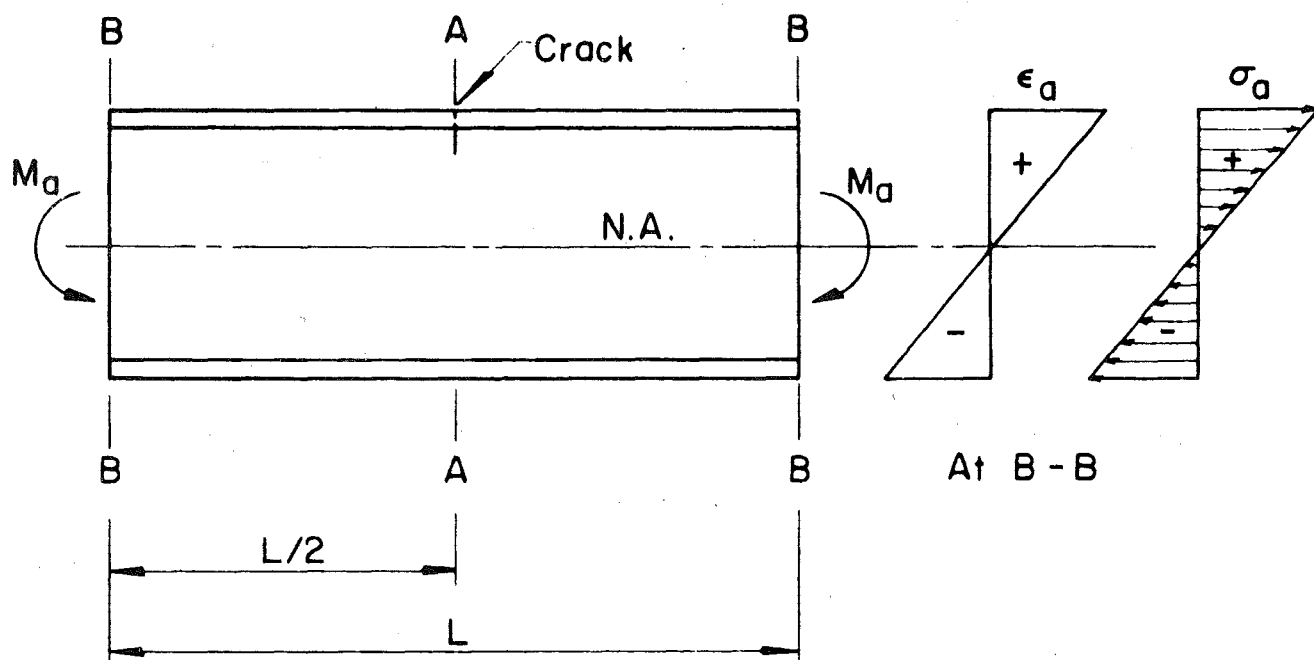


Fig. 1 I-Beam Under Investigation



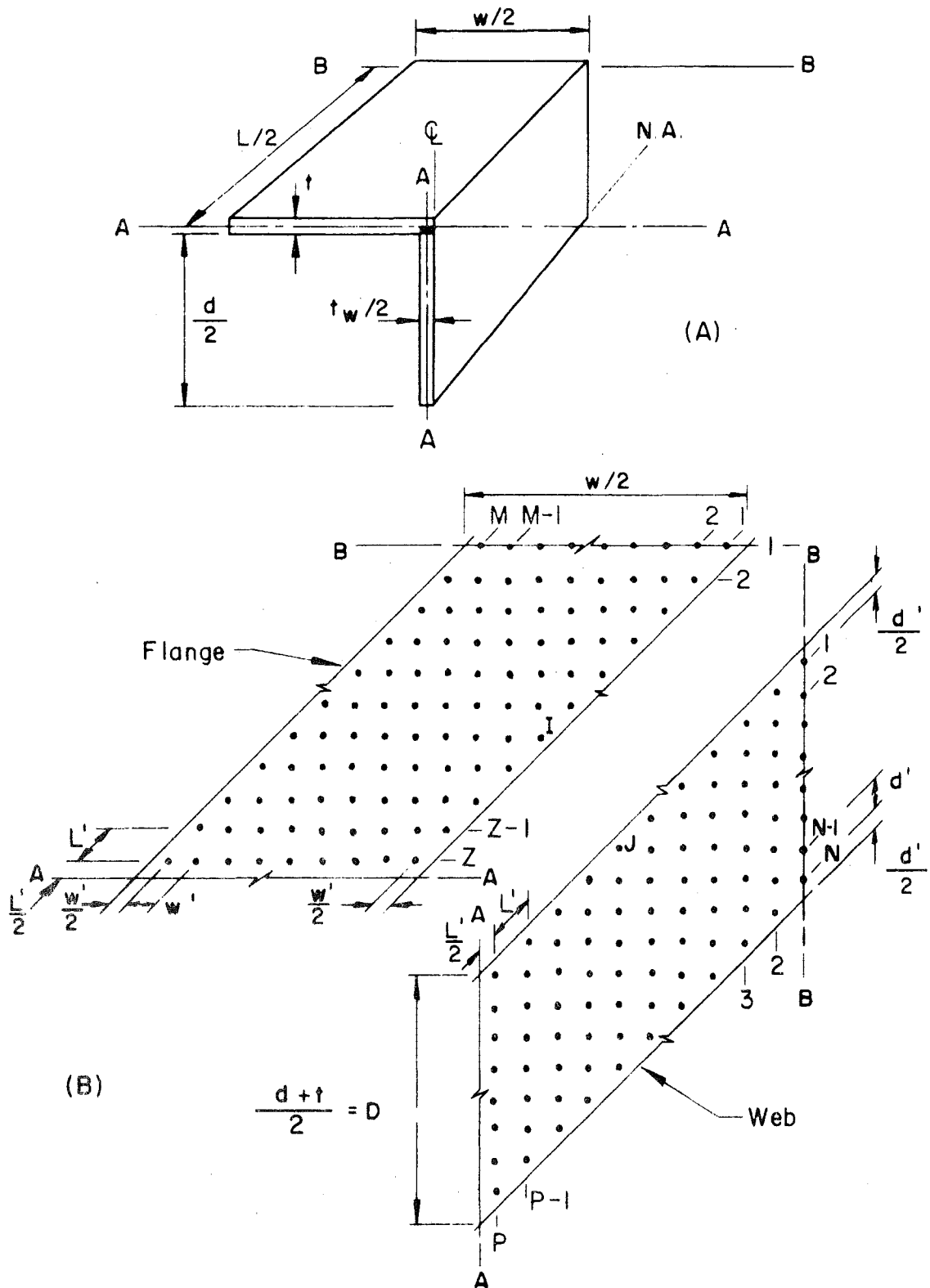
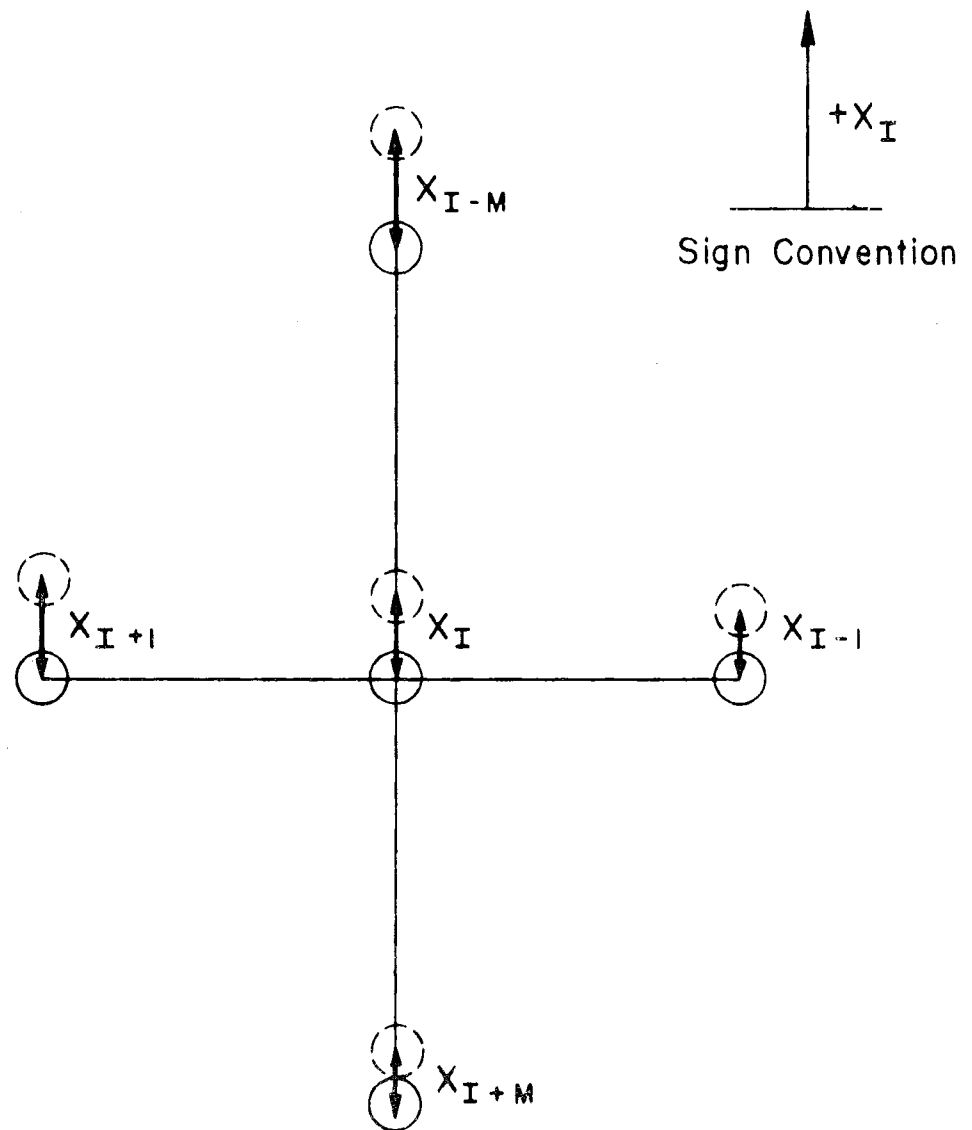


Fig. 2 Discrete Model Representations of the Flange and the Web



$X_I$  = Displacement of Nodal Point From Initial  
Position in Discrete Model

Fig. 3 Typical Displacement Configuration

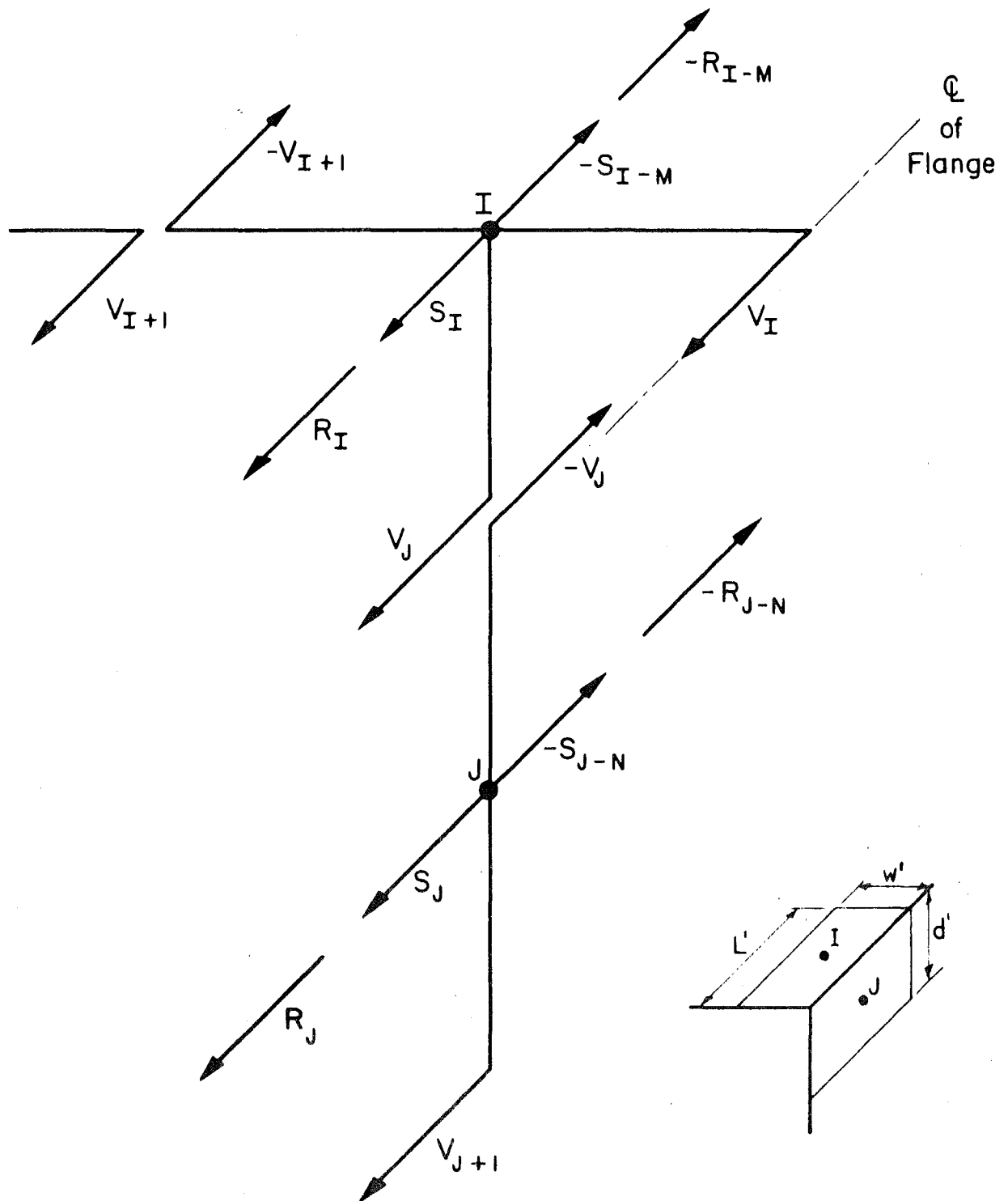


Fig. 4 Stress Resultants of Two Adjacent Elements

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$\sigma_{\max.} = 36.2 \text{ ksi}$

$\sigma_{\min.} = 0 \text{ ksi}$

Mesh Division (20 x 20)

Experimental Data <sup>-31</sup>

|        |            |
|--------|------------|
| 0 kips | 80 kips    |
| ●      | ○ 0 Cycles |
| ▲      | △ 25,000   |
| ■      | □ 40,000   |

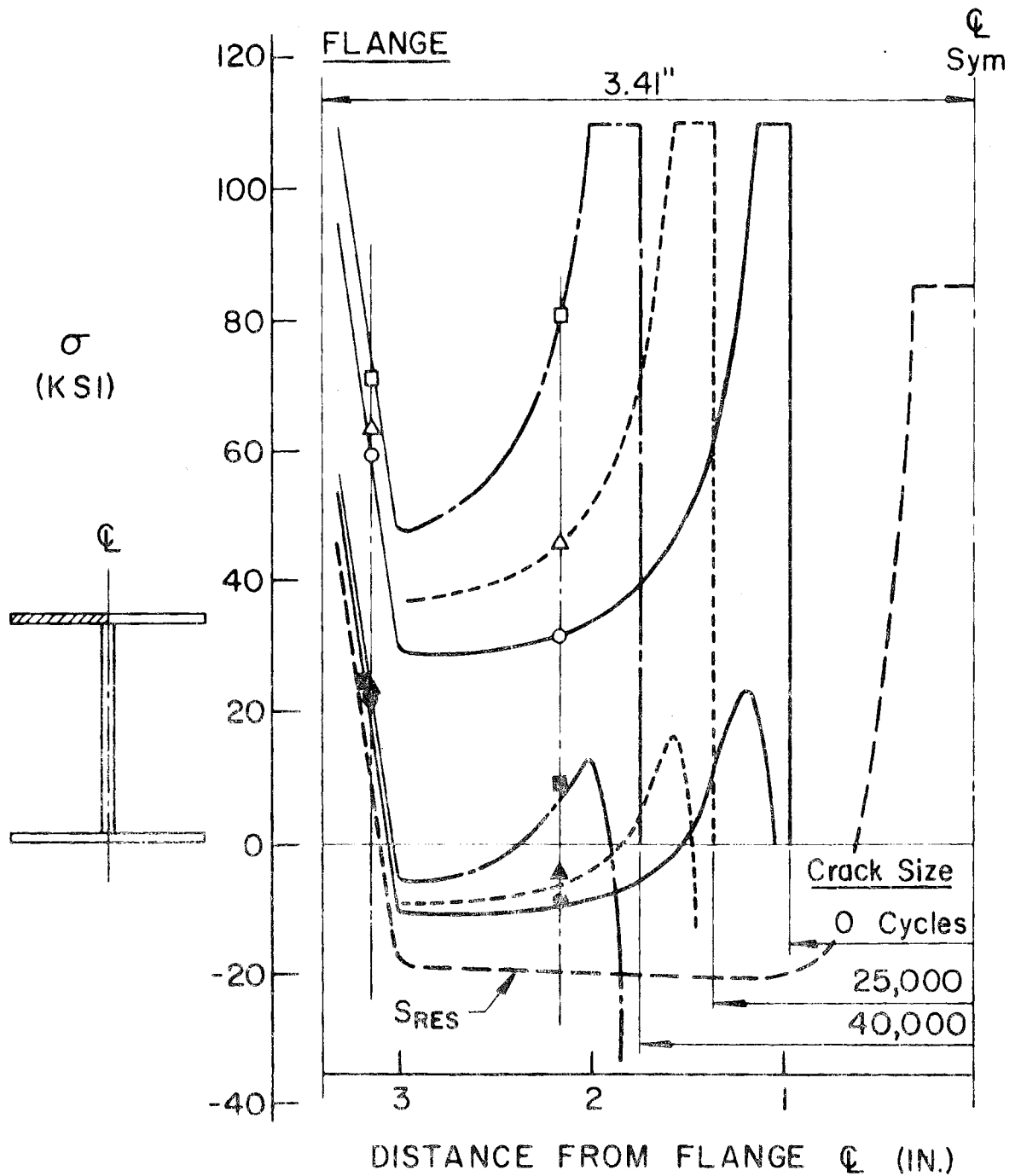


Fig. 5 Theoretical and Experimental Stresses for the Flange

$$\sigma_{\max.} = 36.2 \text{ ksi}$$

$$\sigma_{\min.} = 0 \text{ ksi}$$

Mesh Division (20 x 20)

### Experimental Data

0 kips

80 kips

●

○

0 Cycles

▲

△

25,000

■

□

40,000

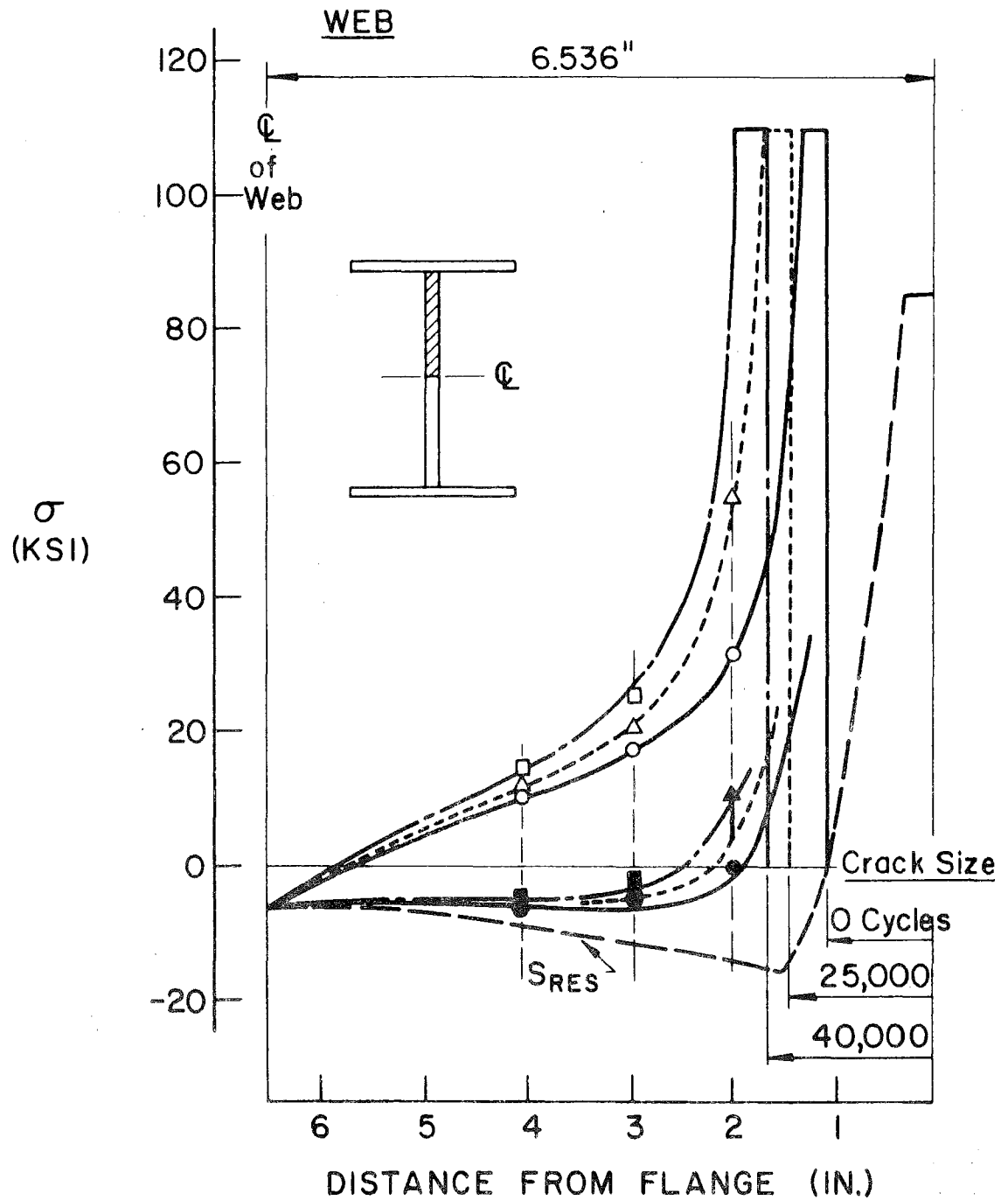


Fig. 6 Theoretical and Experimental Stresses for the Web

$$\sigma_{\max.} = 36.2 \text{ ksi}$$

$$\sigma_{\min.} = 0 \text{ ksi}$$

Mesh Division (20 x 20)

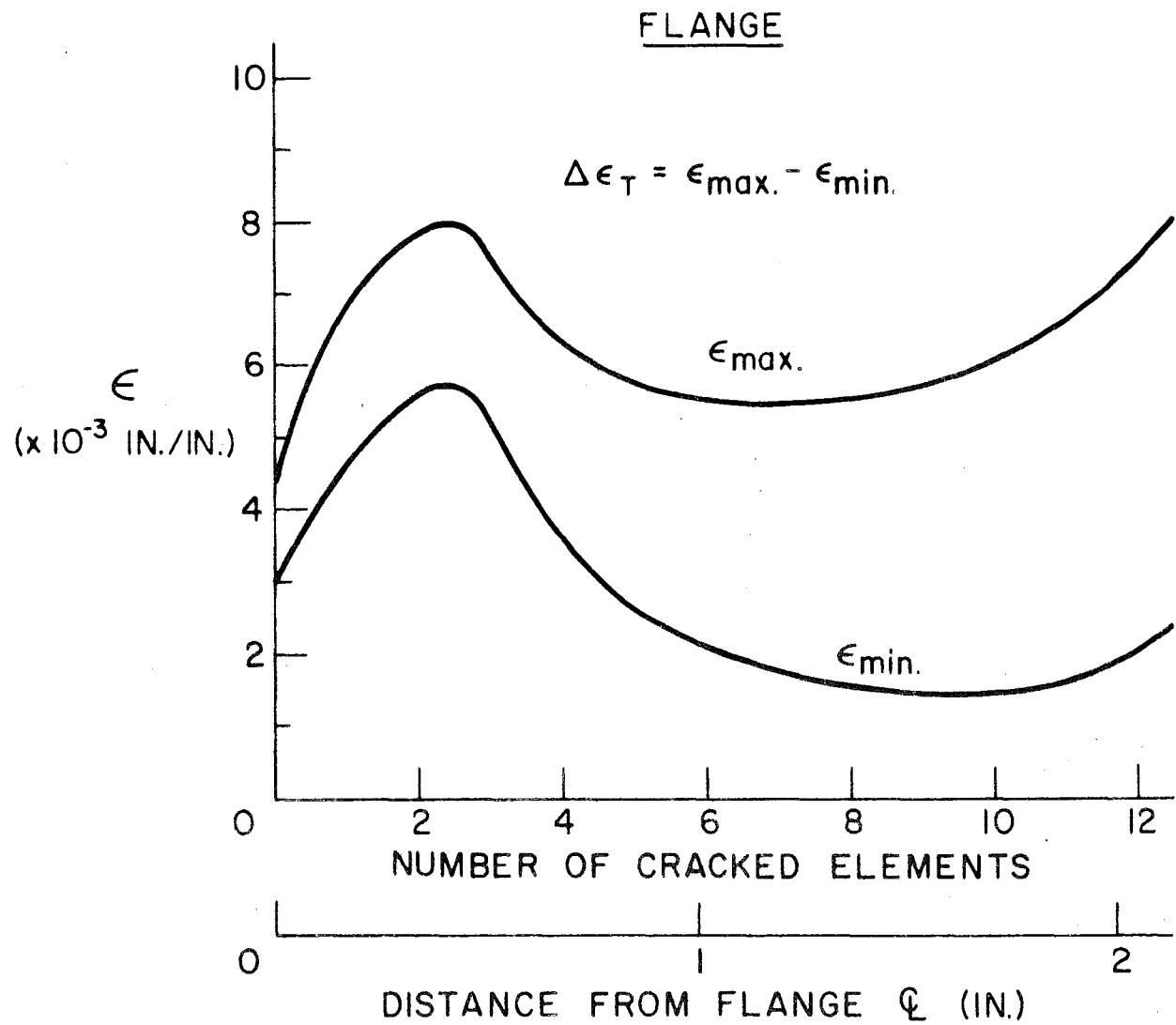


Fig. 7 Strains in the Flange  
Near the Crack Tip

$$\sigma_{\max.} = 36.2 \text{ ksi}$$

$$\sigma_{\min.} = 0 \text{ ksi}$$

Mesh Division (20 x 20)

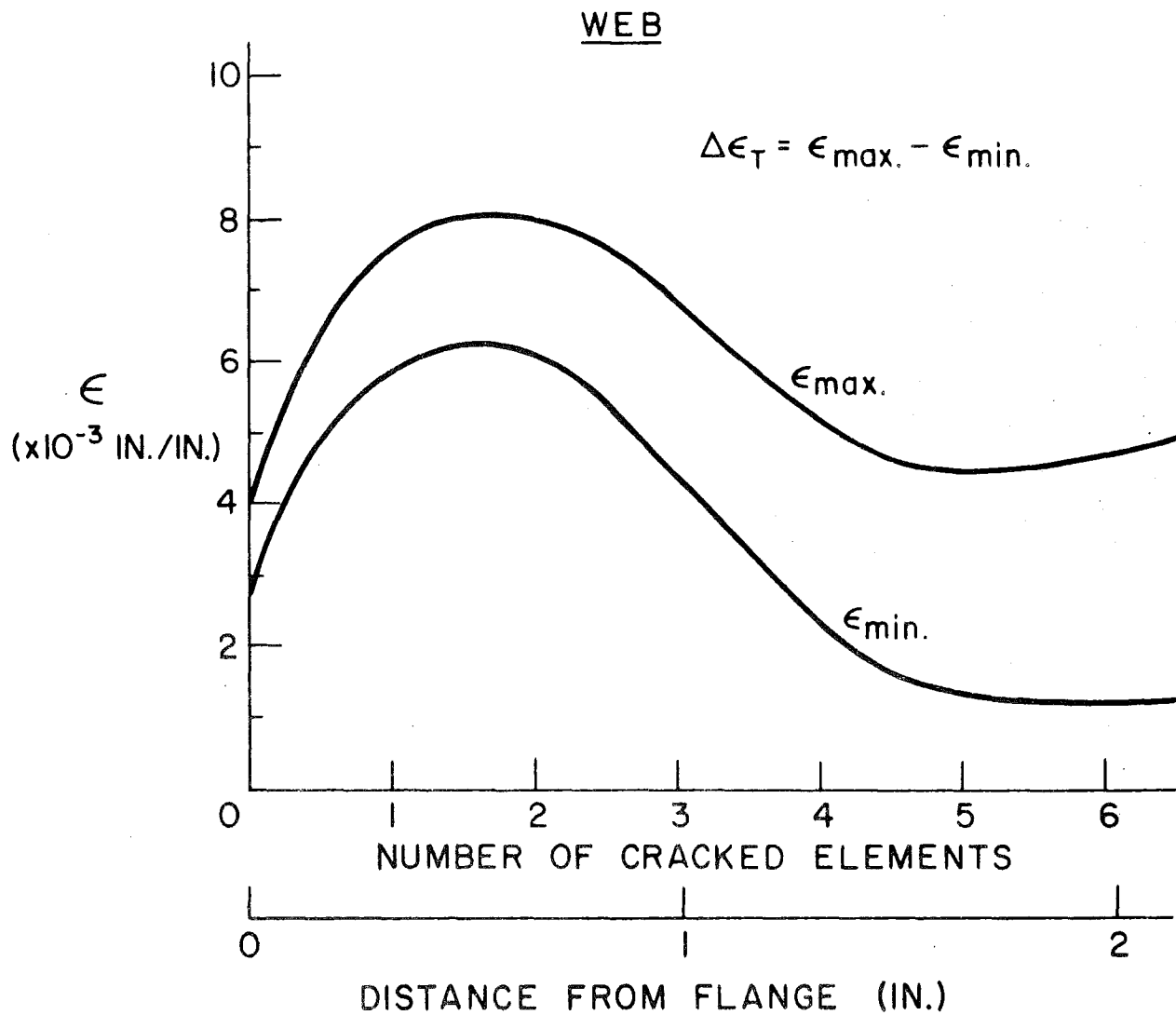


Fig. 8 Strains in the Web  
Near the Crack Tip

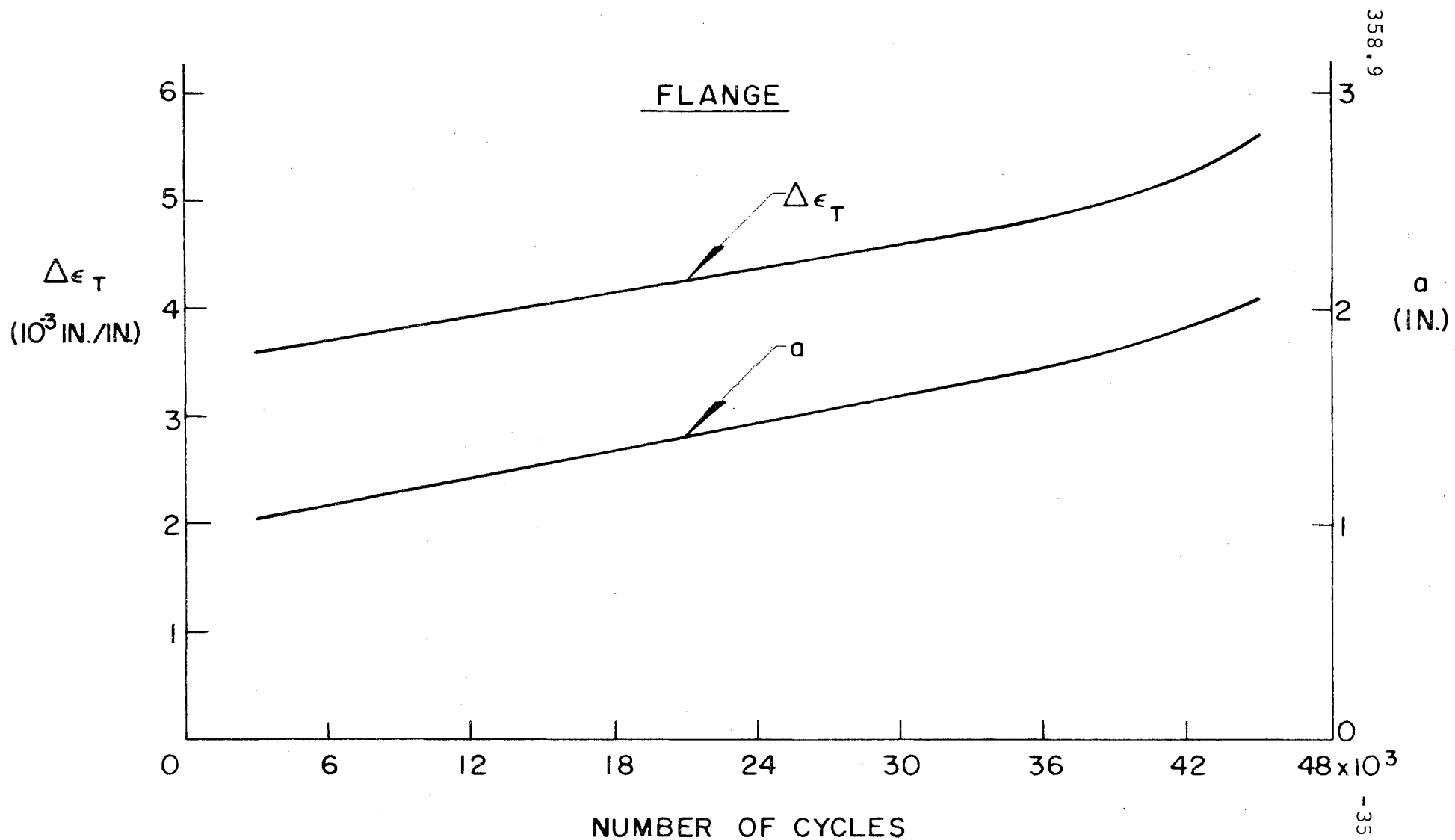


Fig. 9 Strain Range and Crack Length Versus Number of Cycles



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| 13. ABSTRACT<br><p>This report presents an analytical method for determining the stress and strain distribution in a welded I-beam with a three-ended crack. This investigation is part of a major research program on the behavior and design of joined structures subjected to low-cycle fatigue.</p> <p>The analysis is on the plane-stress elastic-plastic behavior of a beam with a crack for one loading-unloading cycle. A mathematical model was developed by replacing the continuous material of the beam by an equivalent system of lumped volumes at discrete points.</p> <p>The results of the discrete element method were compared with the stresses and strains recorded in an experimental investigation of a welded beam with a three-ended crack in the last phase of its fatigue life. Satisfactory correlation between measured and theoretical results was observed.</p> |  |   |                      |